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DETERMINING THE HEAT-TRANSFER CONSTANTS OF AN ORTHOTROPIC LAYER

E. A. Belov, G. Ya. Sokolov, A. S. Starkov, and V. P. Zotov

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We propose a method for the calculation of the principal coefficients of the heat-conduction sensor of a flat orthotropic layer, based on results from a nonsteady experiment.

Methods have been proposed in [1, 2] to determine the heat-transfer parameters of anisotropic materials without destruction of their integrity. The object of the study in this case can only be a massive body which can be assumed to be semibounded from the standpoint of heat. In the following we discuss the possibility of determining the thermophysical characteristics of anisotropic materials in the form of black sheets or of coatings on metal structures.

Let us examine an orthotropic layer in an initial uniform temperature field. Let one of its boundary surfaces be forcibly maintained at a constant temperature, while the other surface is insulated against heat, with the exception of a rectangular area that is acted upon by a source of heat arbitrarily distributed over the surface. Let us introduce into this examination the coordinate system (x, y, z), connected to the orthotropy axes, as is shown in Fig. 1. The temperature field $t(x, y, z, \tau)$ for the layer under these conditions will serve as the solution for a boundary-value problem of the form:

$$\lambda_x \frac{\partial^2 t}{\partial x^2} + \lambda_y \frac{\partial^2 t}{\partial y^2} + \lambda_z \frac{\partial^2 t}{\partial z^2} - c\rho \frac{\partial t}{\partial \tau} = 0, \tag{1}$$

$$t(x, y, z, 0) = 0, \tag{2}$$

$$t(x, y, 0, \tau) = 0, \tag{3}$$

$$\lambda_z \frac{\partial t(x, y, \delta, \tau)}{\partial z} = \begin{cases} q(x, y, \tau), & x, y \in S, \\ 0, & x, y \bar{\in} S. \end{cases} \tag{4}$$

The function $t(x, y, z, \tau)$ tends toward zero at infinity along the coordinates (x, y) together with the first derivatives.

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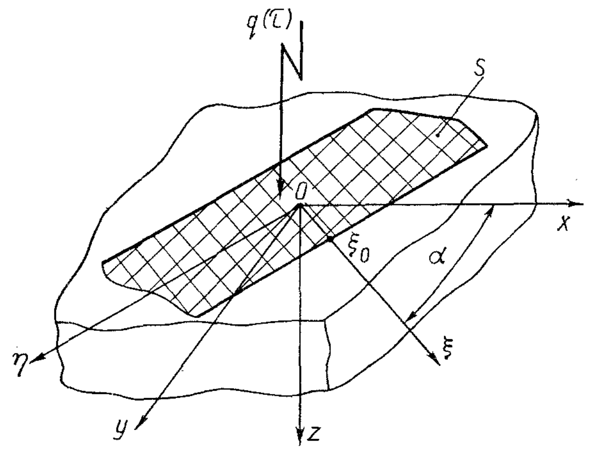

Fig. 1. The thermal diagram of the experiment.

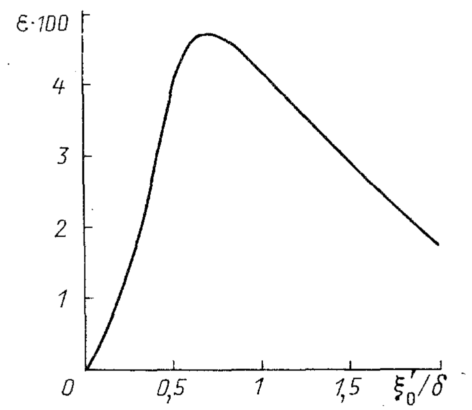

Fig. 2. Error in the determination of the effective heat-source width. ε, %.

We will apply the following integral transformation to problem (1)-(4):

$$\tilde{t}(x,\ y,\ z) = \int_0^\infty t(x,\ y,\ z,\ \tau)\, d\tau. \tag{5}$$

If the energy released by the heat source during the experiment is limited, the temperature fields in the layer over the passage of time relaxes to the original state and the boundary-value problem is transformed as follows:

$$\lambda_x \frac{\partial^2 \tilde{t}}{\partial x^2} + \lambda_y \frac{\partial^2 \tilde{t}}{\partial y^2} + \lambda_z \frac{\partial^2 \tilde{t}}{\partial z^2} = 0, \tag{6}$$

$$\tilde{t}(x,\ y,\ 0) = 0, \tag{7}$$

$$\lambda_z \frac{\partial \tilde{t}(x,\ y,\ \delta)}{\partial z} = \begin{cases} Q(x,\ y),\ x,\ y \in S, \\ 0,\ x,\ y \bar{\in} S, \end{cases} \tag{8}$$

where the function Q(x, y) has the sense of the energy density for the heat source.

Let us change to the coordinate system (ξ, η) connected to the source and derived from the original rotation through some angle α about the z axis:

$$(\lambda_x \cos^2\alpha + \lambda_y \sin^2\alpha) \frac{\partial^2 \tilde{t}}{\partial \xi^2} + 2\sin\alpha\cos\alpha(\lambda_x - \lambda_y) \frac{\partial^2 t}{\partial \xi \partial \eta} +$$
$$+ (\lambda_x \sin^2\alpha + \lambda_y \cos^2\alpha) \frac{\partial^2 \tilde{t}}{\partial \eta^2} + \lambda_z \frac{\partial^2 \tilde{t}}{\partial z^2} = 0. \tag{9}$$

The form of the boundary conditions on transition to the new system obviously does not change.

Let us integrate the derived equation over the coordinate η. We obtain

$$(\lambda_x \cos^2\alpha + \lambda_y \sin^2\alpha) \frac{\partial^2 \hat{\hat{t}}}{\partial \xi^2} + \lambda_z \frac{\partial^2 \hat{\hat{t}}}{\partial t^2} = 0, \tag{10}$$

$$\lambda_z \frac{\partial \hat{\hat{t}}\,(\xi,\ \delta)}{\partial z} = \begin{cases} \hat{Q}\,(\xi), & \xi \leqslant \xi_0, \\ 0, & \xi > \xi_0. \end{cases} \tag{11}$$

Let us undertake a scale transformation of the coordinate ξ according to $\xi' = \xi/\mu_1$, where

$$\mu_1 = \left(\frac{\lambda_x \cos^2\alpha + \lambda_y \sin^2\alpha}{\lambda_z} \right)^{1/2}. \tag{12}$$

As a result we obtain a two-dimensional isotropic boundary-value problem defined by the relationships

$$\frac{\partial^2 \hat{\hat{t}}}{\partial \xi'^2} + \frac{\partial^2 \hat{\hat{t}}}{\partial z^2} = 0, \tag{13}$$

$$\lambda_z \frac{\partial \hat{\hat{t}}\,(\xi',\ \delta)}{\partial z} = \begin{cases} \hat{Q}\,(\xi'), & \xi' \leqslant \xi_0', \\ 0, & \xi' > 0. \end{cases} \tag{14}$$

In the following we will limit ourselves to the special case of uniform distribution of the evolved heat over the surface of the source. The solution of problem (13), (14) can be written with the aid of the quadratures

$$\hat{\hat{t}}\,(\xi',\ \delta) = \frac{\hat{Q}}{\lambda_z} \frac{2}{\pi} \int\limits_0^{+\infty} \frac{\operatorname{th} p\delta}{p^2} \sin p\xi_0' \cos p\xi' dp. \tag{15}$$

Let us note that the integral transform of temperature, as determined from relationship (15), is a monotonically ascending function of the effective width ξ_0' of the heating region. If we know the quantities $\hat{Q}$ and λ_Z, having measured the temperature at some point at the surface $z = \delta$, we can find the effective width ξ_0' of the source and, consequently, we can determine the coefficient of scale transformation μ_1. After we have completed the second measurement, having turned the source through an angle of 90° about the z axis, for the scale coefficient we obtain the value of μ_2:

$$\mu_2 = \left(\frac{\lambda_x \sin^2\alpha + \lambda_y \cos^2\alpha}{\lambda_z} \right)^{1/2}. \tag{16}$$

Having jointly solved Eqs. (12) and (16), it is not difficult to express the coefficients of the heat-conducting tensor λ_x and λ_y in terms of λ_z, μ_1, μ_2, and α. For example,

$$\lambda_x = \lambda_z \frac{\mu_1^2 \cos^2\alpha - \mu_2^2 \sin^2\alpha}{\cos 2\alpha}. \tag{17}$$

If the angle α, rounding off the orientation of the orthotropy axes relative to the heat source, is not known in advance, it becomes necessary to carry out a third measurement, turning the source about the z axis through an angle of 45° relative to its initial orientation. We then obtain

$$\mu_3 = \left(\frac{\lambda_x \cos^2(\alpha + 45^\circ) + \lambda_y \sin^2(\alpha + 45^\circ)}{\lambda_z} \right)^{1/2}. \tag{18}$$

A system of three equations for μ_1, μ_2, and μ_3 makes it possible to find the angle

$$\alpha = \frac{1}{2} \operatorname{arctg} \frac{2\mu_3^2 - \mu_1^2 - \mu_2^2}{\mu_1^2 - \mu_2^2}. \tag{19}$$

If during the measurements it turns out that $\mu_1 = \mu_2$, then it follows from relatoinships (12) and (16) that $\alpha = 45°$ and formulas (17) and (18) turn out to be unsuitable. In this case, we can recommend the formulas

$$\lambda_x = \lambda_z (2\mu_1^2 - \mu_3^2) \tag{20}$$

and

$$\lambda_y = \lambda_z \mu_3^2. \tag{21}$$

Let us take note of the fact that the method with which to find the coefficient λ_z is well known [3] and can be employed in any ITNR series apparatus [4]. Thus, in order to find the coefficients λ_x and λ_y, we have to determine the integral transform of the surface temperature of the layer and to establish an algorithm to find the effective width of the source in accordance with relationship (15). Let us dwell on these questions in somewhat greater detail. Starting at some instant of time τ_r, the temperature at any point on the surface diminishes exponentially. This allows us to determine the integral in (15) on the basis of the experimental temperature values in the finite interval $[0, \tau_r]$ and the steady-state value for its rate of change:

$$\int_0^\infty t(x, y, \delta, \tau)\, d\tau = \int_0^{\tau_r} t(x, y, \delta, \tau)\, d\tau + \frac{t(x, y, \delta, \tau_r)}{m}, \tag{22}$$

where $m = -t^{-1} dt/d\tau$.

For the point at which to record temperature in finding the effective source width ξ_0', and the parameters μ_1, μ_2, μ_3 on that basis, it is expedient to use the surce center $\xi' = 0 = \xi$. Here the integral in (15) is calculated explicitly:

$$\hat{t}(0, \delta) = \frac{\hat{Q}}{\lambda_z}\left[1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi\xi_0'}{2\delta}\right)\sum_{k=0}^{\infty}\frac{\exp-(\pi k\xi_0'/\delta)}{(2k+1)^2}\right]. \tag{23}$$

In order to find ξ_0' from (23) we can recommend an approximate expression of the form

$$\hat{t}(\xi_0') = \frac{\hat{Q}}{\lambda_z}\left[1 - 0{,}94172 \exp\left(-\frac{\pi\xi_0'}{1{,}605\delta}\right)\right]. \tag{24}$$

The approximation error ε, defined as $\hat{t}(0, \delta) - \hat{t}(\xi_0')/\hat{t}(0, \delta)$, is shown in Fig. 2 for various source dimensions.

Determination of the integral on the basis of the temperature coordinate η can be accomplished, for example, by means of a resistance thermometer whose current-conducting elements are positioned along the ξ axis. However, for an elongated source $\xi_0 >> \eta_0$ (approximation of an infinite strip) the relationship between the temperature field and the coordinate η is absent, the equation of heat conduction coincides in form with (10), and there is no need to determine the integral temperature. In conclusion, let us note that the proposed method can be recommended both for laboratory testing of specimens of anisotropic materials and for nondestructive monitoring of coatings on metal structures.

NOTATION

x, y, z, ξ, η, coordinates; δ, thickness of layer; $t(x, y, z, \tau)$, temperature at a point in the material having coordinates x, y, z at the instant of time τ; λ_x, λ_y, and λ_z, principal coefficients of the heat-conduction tensor; c, heat capacity; ρ, density of the material; q, density of heat flux; Q, energy; α, angle of coordinate system rotation; m, rate of material point cooling.

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